# Rivalry and the excessive allocation of resources to research

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*This article presents a simple probability model of R&D which suggests that competitive firms may overinvest resources in research, even in the face of uncertainty, inappropriability, and increasing costs of research. In the presence of uncertainty, some duplication of R&D efforts may be justified because of the increased probability of success that results, but competitive equilibria may be characterized by excessive duplication. Further, when different firms can discover different things, excessive knowledge may be produced, even when each firm individually performs less R&D than is socially desirable. This is a consequence of excessive entry.*

## 1. Introduction

■ This article seeks to examine conditions under which a competitive system might lead to overinvestment of resources in research activity. In an influential paper, Arrow (1962) pointed out that three of the classic reasons for market failure in resource allocation applied to the problem of invention: indivisibilities, inappropriability, and uncertainty. Each of these factors creates a bias for the competitive system to allocate to research fewer resources than is socially optimal.

More recently, however, a number of authors have noted that there exists a fourth reason for market failure in this area—the lack of property rights in *potential* inventions.[1] Undiscovered knowledge is viewed as a common resource, and it is well known[2] that allocation of resources in such cases is not efficient. Briefly, an excessive amount of the common resource is used because use is carried to the point where *average* revenue, rather than marginal revenue, equals marginal cost. Thus, there may be a tendency for firms to perform excessive R&D, or for a "rush to invent" which dissipates much of the social benefit from research. Whether there is actually excessive or insufficient R&D then depends on the relative magnitudes of this effect and of the forces pointed to by Arrow.

This point is now well known.[3] This article presents a simple probability model that

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[1] Recently, Kitch (1977) has argued that, in fact, patents are precisely that—property rights in *potential* inventions. He calls them prospects, since he views them as similar to mineral claims. This does not remove the problem, however, but merely shifts it to a more primitive level. There are still no property rights in potential prospects. For a more general discussion of the phenomenon of rent-seeking, and the possibility of dissipation of benefits, see Buchanan *et al.* (1980).

[2] The literature here is quite large. The usual references are Gordon (1954) and Smith (1968, 1969). See also Weitzman (1974).

[3] The most complete discussion is found in Dasgupta and Stiglitz (1980a, 1980b) and Nelson (1979). Other work that has played a role here includes Barzel (1968), Hirshleifer (1971), Kamien and Schwartz (1972), Loury (1979), and Lee and Wilde (1980).

captures this basic effect, while highlighting the role that uncertainty and inappropriability might play in the common pool problem.[4] The model views research as a process of random sampling. In this sense, it is more in the spirit of the R&D models of Nelson (1961), Marschak, Glennan, and Summers (1967), Weitzman (1979), and Roberts and Weitzman (1981). It adds to this line of literature an examination of free entry, thereby making market structure endogenous,[5] and a comparison of the free entry outcomes with socially efficient ones.

Several interesting results emerge. First, the possibility of overinvestment of resources in research is clearly shown. Second, it is shown that, in the presence of uncertainty, duplication of research effort is not necessarily wasteful.[6] When research outcomes are uncertain, it may be socially desirable to run several lines of enquiry simultaneously, even if *ex post* they may turn out to have been duplicative. This results because the *ex ante* probability of overall success may be increased by duplication. However, it is seen that competition could lead to an excessive amount of duplication. Third, it is argued that the amount of information generated (or the size of cost-reducing innovations) may also be excessively large.[7]

Section 2 presents the basic model, where firms are restricted to follow only one research project at a time. Free entry is allowed, however, so that the market structure of this industry is endogenous. Section 3 extends the model to allow firms to pursue several projects, and Section 4 examines the case of increasing costs.

## 2. The basic model of R&D as a sampling problem

■ Research is viewed here as a sampling process. Suppose $n$ firms have decided to undertake research in a particular industry. Each firm is assumed to face the same technological opportunities, and these are assumed to be invariant to the number of firms searching. The opportunities are characterized in the following way: there are $N$ possible lines of research, each indistinguishable *ex ante* from the others. *Ex post,* however, the lines of research can be divided into two types. There are $S$ ($<N$) "strikes," i.e., fruitful lines of research, while the rest are dry holes. The dry holes produce nothing of value. Each strike generates a social payoff normalized to unity and a privately appropriable payoff $g < 1$. This simple model captures the fact that R&D is characterized by uncertainty and that the results are only partially appropriable. The latter, however, is only part of the public goods character of R&D. It could be argued that the assumption of independence in the payoffs to research is unrealistic, since the value of what one firm discovers depends intimately on what others may have discovered. I have considered elsewhere a model where the social payoff is a concave function of the number of strikes (Tandon, 1978, ch. 2), but do not report the results here since they do not change qualitatively, while there is some loss of simplicity.

Research proceeds in the following way. Each firm that wants to "play" pays an amount $c$ and randomly chooses one of the lines of research. No firm knows which lines of research others have chosen, but firms do know how many other firms are playing, and that each one is choosing randomly. If only one firm selects a given strike, it of course receives the payoff $g$. If more than one firm chooses a given line that results in a strike,

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[4] For a discussion of some uncertainty issues relating to another common resource—petroleum—see Stiglitz (1975) and Peterson (1975).

[5] There is a growing view that this is the appropriate assumption. Apart from Dasgupta and Stiglitz, see Schumpeter (1947), Nelson and Winter (1977), Loury (1979), and Futia (1980). For a promising start to empirical work in this area, see Levin (1980).

[6] This point was noted by Nelson (1961) but has received insufficient attention.

[7] This contradicts the conclusion of Dasgupta and Stiglitz (1980b). The key difference is that they do not consider uncertainty when discussing the size of cost reductions.

I assume that only one receives the payoff $g$. This corresponds to the normal assumption that the *first* to succeed gets the payoff, either because it receives a patent or because imitation lags give it an advantage. The present model, however, is timeless, so that I assume instead that if $x$ firms hit the same strike, one among them is chosen randomly for the patent. In other words, in case of a tie, the order of finish is determined randomly, with the first firm getting all the benefits. This assumption is not necessary, but does make the analysis considerably more tractable and intuitive. I discuss briefly at the end of this section the effect of assuming $g$ to be a declining function of $x$.

□ **The firm's problem.** Free entry is assumed. If firms are risk neutral, entry will occur as long as expected net profits are positive. An individual firm has a chance $S/N$ of hitting a successful project. The probability that $x$ other firms hit the *same* strike is $\binom{n-1}{x} p^x q^{n-1-x}$, where $p = 1/N$ and $q = 1 - p$. Thus the expected revenue to the typical firm[8] is

$$E(R) = \left(\frac{Sg}{N}\right) \sum_{i=0}^{n-1} \binom{n-1}{i} p^i q^{n-1-i} \left(\frac{1}{i+1}\right).$$

Reindexing, we get

$$E(R) = \left(\frac{Sg}{N}\right) \sum_{i=1}^{n} \binom{n-1}{i-1} p^{i-1} q^{n-i} \left(\frac{1}{i}\right).$$

But $\binom{n-1}{i-1} = \binom{n}{i} \frac{i}{n}$. Thus, we have

$$E(R) = \left(\frac{Sg}{N}\right) \sum_{i=1}^{n} \binom{n}{i} p^{i-1} q^{n-i} \left(\frac{1}{i}\right)\left(\frac{i}{n}\right) = \left(\frac{Sg}{N}\right)\left(\frac{1}{np}\right) \sum_{i=1}^{n} \binom{n}{i} p^i q^{n-i}.$$

But $\binom{n}{i} p^i q^{n-i}$ is the binomial probability of $i$ "hits." Thus, we have the following expression for expected profits:

$$E(V) = \left(\frac{Sg}{N}\right)\left(\frac{1}{np}\right)\left\{1 - \binom{n}{0} p^0 q^n - c\right\} = \left(\frac{Sg}{n}\right)(1 - q^n) - c \tag{1}$$

since $p = 1/N$.

Under free entry, the equilibrium number of firms, $n_e$, will be the value of $n$ that sets $E(V)$ equal to zero. Thus

$$gS(1 - q^{n_e}) = cn_e. \tag{2}$$

Of course, since $n_e$ must be an integer, this equation may not be satisfied. For the sake of simplicity, the integer problem will be suppressed and $n$ will be treated as a continuous variable. Note that (2) represents the typical common-pool equilibrium, where average revenue must equal average costs.

Equation (2) yields an economically meaningful solution only if $n_e$ is positive. If, however, net returns are always negative, $n_e$ will be zero. Thus,

$$n_e = 0 \qquad \text{if} \qquad c > \lim_{n \to 0} \frac{Sg}{n}(1 - q^n).$$

Using l'Hospital's Rule, we may write

[8] I would like to thank an anonymous referee for suggesting this particular derivation for the firm's expected profits, which is clearer than the original.

$$n_e = 0 \qquad \text{if} \qquad c > Sg(-\ln q) \tag{3}$$

and $n_e$ is the solution to (2) otherwise.

Expression (2) summarizes the forces that jointly determine market structure and levels of R&D spending in this industry. Simple comparative statics exercises reveal that more firms will enter if costs of research $c$ fall, if the technological opportunities $S$ rise, and if the degree of appropriability $g$ rises. Note that this simple expression summarizes the arguments for two of the major determinants of R&D spending. The "technological opportunities" argument associated with Rosenberg (1974) is captured by the effects of $c$ and $S$ on $n_e$, while the market structure argument associated with Schumpeter (1947) and Galbraith (1956) may be summarized by the appropriability parameter $g$. The role of demand inducement, associated with Schmookler (1966) has been suppressed (or subsumed in $g$) since each strike has been assumed to yield the same social and private payoffs.

□ **The social problem.** There are several ways in which the social problem could be posed. If society could run all research activity sequentially, it would keep sampling as long as the cost of sampling the next project was less than the expected social benefit. Alternatively, if it were possible to identify the research lines *ex ante,* society could choose exactly $N$ firms—one for each research line—or none at all. Neither of these views is really comparable to the firms' problem. I shall, therefore, pose the social problem as one where society simply provides funds for the research performed by firms, but does not control the research content in any way. It will be assumed that firms do not communicate with each other. Thus, each firm will behave just as before; only now the number of firms that enter will be a choice variable for the government. The same model could describe an optimal licensing procedure, or it could be thought of as a way of finding the optimal market structure under blocked entry.

Suppose that $n$ firms have been chosen to perform research. Each one is sampling randomly as before. Although this sampling occurs simultaneously, it is analytically more convenient to think of the social R&D process as a sequence of $n$ random, independent drawings with replacement, with the probability of success falling continually. The $n$ firms are randomly ordered, and then their "success" is examined in order. In this experiment, "success" means making a *new* strike, i.e., a strike not made by any firm higher in the order. If the probability of success is adjusted on each iteration, the successive trials may be regarded as independent; then the expected number of strikes will merely be the sum of the expected strikes on successive trials.

Suppose the probability of a success on the $(i - 1)$th trial is $P_{i-1}$. Consider now the probability $P_i$. If a strike was achieved on the $(i - 1)$th trial, there will be one less potential strike on the $i$th trial. Hence $P_i$ would have fallen by $1/N$. If, on the other hand, the $(i - 1)$th trial had hit a dry hole or a previously made strike, the probability of a strike on the next trial would be unchanged. Therefore,

$$P_i = P_{i-1}\left(P_{i-1} - \frac{1}{N}\right) + (1 - P_{i-1})P_{i-1}$$

or

$$P_i = P_1\left(1 - \frac{1}{N}\right)^{i-1} = P_1 q^{i-1}.$$

Now clearly $P_1 = S/N$. Therefore,

$$\sum_{i=1}^{n} P_i = S(1 - q^n). \tag{4}$$

This is the expected gross social payoff, since for each strike the social payoff is unity. Note that this is simply the per firm gross private payoff multiplied by $n$ and divided by $g$. The net social payoff, therefore, is

$$W(n) = S(1 - q^n) - cn. \tag{5}$$

Treating $n$ as a continuous variable, society will attempt to maximize $W(n)$. Now

$$W'(n) = -Sq^n \ln q - c$$

and

$$W''(n) = -Sq^n(\ln q)^2 < 0.$$

Thus $W(n)$ is concave and there will be a maximum at $W'(n) = 0$. Therefore, the social optimum $n^*$ satisfies

$$q^{n^*} = \frac{c}{-S \ln q}. \tag{6}$$

Since $q^n$ declines as $n$ increases, it is easy to see that $n^*$ rises as $c$ falls or as $S$ rises. Both effects are intuitively obvious.[9]

Now define

$$b = \frac{c}{-S \ln q}. \tag{7}$$

Then

$$n^* = \frac{\ln b}{\ln q}. \tag{8}$$

This expression will be useful for the proposition below.

□ **Comparison of the free-entry equilibrium and the social optimum.** Comparison of $n_e$ with $n^*$ is made with the aid of Figure 1. The social optimum $n^*$ occurs where the *marginal* social benefit equals marginal social cost. The free-entry equilibrium $n_e$ occurs where *average* private benefit equals average (and marginal) cost. The private benefit curve $gS(1 - q^n)$ is simply a fraction $g$ of the social benefit. Under full appropriability, i.e., with $g = 1$, there will clearly be excessive entry in the competitive equilibrium. This is represented by $n^o$, which will always be to the right of $n^*$. Whether $n_e$ is greater or less than $n^*$ depends upon the degree of appropriability $g$. One way to express the precise condition on $g$ that determines how $n_e$ and $n^*$ are related is to note whether, at $n = n^*$, total private benefit is greater or less than total cost. Clearly

$$n_e > n^* \qquad \text{iff} \quad gS(1 - q^{n^*}) > n^*c.$$

That is, $n_e > n^*$ iff $g > \dfrac{n^*c}{S(1 - q^{n^*})}$.

But, using (6), (7), and (8), this inequality can be written

$$g > \frac{-b \ln b}{1 - b}.$$

We, therefore, have the following complete list of possibilities.

*Proposition 1:*

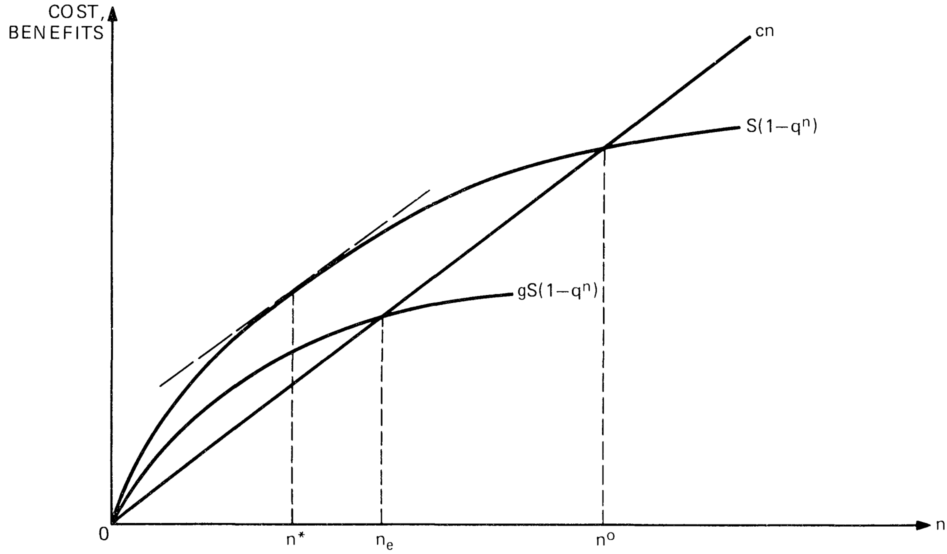
(i) If $g < b$, no firms will enter; this is suboptimal.

(ii) If $b < g < \dfrac{-b \ln b}{1 - b}$, $n_e < n^*$, i.e., there will be underinvestment of resources in research.

[9] Arditti and Levy (1980) have extended this model to examine the effects of uncertainty in payoffs on $n^*$.

FIGURE 1

SOCIAL AND PRIVATE SOLUTIONS TO THE BASIC MODEL




(iii) If $g > \dfrac{-b \ln b}{1 - b}$, $n_e > n^*$, i.e., there will be overinvestment in research.

Note that (i) follows directly from (3) and the definition (7).

This proposition establishes conditions for over- and underinvestment in research. Note that the conditions are quite reasonable. It is easy to show that the expression $\dfrac{-b \ln b}{1 - b}$ increases monotonically with $b$ and that $b$ rises as $c$ rises or $S$ falls. Thus, the possibility of overinvestment increases: as $g$, the degree of appropriability, increases; as $c$, the cost of research, falls; and as $S$, a measure of technological opportunities, increases. Note that in this model, $n^*$ may be greater than one, i.e., in the presence of uncertainty, it may be socially optimal to run several research lines simultaneously. Thus, duplication of research effort is not inherently wasteful; we may even speak of an optimal amount of duplication or redundancy. Overinvestment of resources now occurs only when the amount of duplication exceeds the optimal amount.

The proposition has an important implication also for the production of knowledge or the size of cost reductions. Each strike may be interpreted as an extra piece of knowledge or a unit cost reduction. Thus the size of the cost reduction would simply be the total number of strikes. The expected number of strikes is given by (4), and it is easy to see that this increases monotonically with $n$. Thus if $n_e > n^*$, the expected number of strikes (or cost reduction) will exceed the social optimum. This reverses the result derived by Dasgupta and Stiglitz (1980b) for a model in which there is no uncertainty. In the absence of uncertainty, all duplication is wasteful, and no individual firm spends more than the social optimum. Since all firms are working on the same technological frontier, this means the cost reductions will always be insufficient. Under uncertainty, however, excessive entry automatically means that the expected number of strikes is also excessive. Thus, the size of the cost reduction may be excessively large. The result is essentially similar to the one in the literature on timing of innovations, where the free-entry introduction date may be premature (Barzel, 1968; Kamien and Schwartz, 1972; Tandon, 1978; Loury, 1979; Dasgupta and Stiglitz, 1980a).

□ **Some alternative assumptions.** The model of this section has involved several restrictive assumptions. They are not all realistic but they do keep the model simple. I shall comment briefly on the effects of altering three of these assumptions.

First, it was assumed that firms make independent draws. A technological determinist might argue that even though firms cannot distinguish *a priori* between research lines, there will be a tendency for them to cluster around certain ideas. Thus, the probabilities of making the same strikes may be correlated. Another way of looking at this kind of problem has been examined by Nelson (1961) and Marschak, Glennan, and Summers (1967). It may be that the costs of research are uncertain but correlated across firms. Nelson, for example, argued that if project costs are correlated, the optimal number of research lines would be smaller than if costs were not correlated. Similarly, in the present model, it is clear that if firms' draws are correlated,[10] duplication is no longer so desirable, and, therefore, the possibility of overinvestment in competitive equilibrium is increased. There will also now be a greater possibility that the size of cost reductions will be insufficient.[11]

Second, it has been assumed that the social payoff to each strike is the same and independent of the number of strikes. Since I am assuming that all these research lines are in the same general area, this assumption seems unrealistic. Specifically, we would expect the social payoff to each successive strike to decline because some of the research results would overlap or because two strikes may actually just represent different ways of achieving the same general goal. Once again, I argue that relaxing this assumption will not alter the basic thrust of the model. The case of diminishing social returns to "strikes" is considered briefly elsewhere (Tandon, 1978, ch. 2), and its dual, the case of increasing costs, is considered in Section 4 below.

Third, it was assumed that the degree of appropriability, $g$, was a constant. If many firms "succeed," this is realistic only if one firm gets a patent and there is no possibility of "inventing around" the patent. Actually, we would expect the degree of appropriability to fall as more entry takes place. The case for overinvestment would then be somewhat weakened. It turns out that explicit analysis of this case is somewhat intractable, and I leave it here as a possible avenue for future work.

## 3. A model with varying sample sizes

■ So far, each firm has been allowed to sample only once. The present section examines the outcome when each firm can choose its sample size. It is assumed that each firm pays $c$ for each research line it decides to sample; further, each strike yields a social payoff of 1 and a private payoff $g$, accruing to a randomly selected patentee from among those firms that make that strike. It is assumed, in this timeless model, that firms must announce their sample sizes in advance.[12]

Several different assumptions could be made with regard to firm behavior. A Cournot-Nash assumption would be that each firm believes the sample sizes of its competitors are fixed and chooses its own sample size under this assumption. In this case, it can be shown that if it is profitable for the firm to sample at all, it will choose to sample the entire population of research projects.

Suppose there are $n$ firms sampling, and we are interested in the sample size, $y_1$, of the firm indexed by subscript 1. Let the sample sizes of the other firms be

[10] Of course, it is assumed that firms are not aware of this, i.e., they believe their draws are independent.

[11] For a fuller discussion of the effects of interdependence on R&D, see Dasgupta and Stiglitz (1980a).

[12] For analysis of the case where firms perform R&D sequentially, and therefore can sample sequentially, see Roberts and Weitzman (1981) and Spulber (1980). This line of work looks only at one firm's problem (or the problem of a socially managed research laboratory). There is no examination of entry or comparison of social with market outcomes. For sequential R&D problems in a game-theoretic setting, see Reinganum (1981).

$y_2, y_3, \ldots, y_n$. Consider the expected payoff to firm 1 from any one of the $S$ potential strikes. The probability that firm 1 makes this strike is $y_1/N$. Let $v$ be the random number of other firms that also make this strike. Then the gross expected payoff to firm 1 from this strike is

$$E(R_1) = \frac{1}{N} y_1 \left\{ gP(v = 0) + \frac{g}{2} P(v = 1) + \ldots + \frac{g}{n} P(v = n - 1) \right\}. \tag{9}$$

Now the probabilities $P(v = j)$ depend only on $y_2, \ldots, y_n$; they certainly do not depend on $y_1$. Thus, it is possible to write

$$E(R_1) = \frac{1}{N} y_1 g V_1,$$

where $V_1 > 0$ is a function of $y_2, \ldots, y_n$. Then the net expected payoff to firm 1 from the whole sample is

$$E(\Pi_1) = \frac{1}{N} y_1 g S V_1 - c y_1. \tag{10}$$

The firm will choose its sample size $y_1$ to maximize its net expected profit. Now

$$\frac{\partial E(\Pi_1)}{\partial y_1} = \frac{gSV_1}{N} - c.$$

This is positive if $c < gSV_1/N$ and negative otherwise; its sign does not depend on the choice of $y_1$. Thus, if it is profitable for the firm to sample even once, its expected net payoff will continue to rise as its sample size rises. In this case, it will sample the entire set of research lines.

Of course, the size of the net expected payoff will depend on the number of firms sampling. Suppose we know that any firm that decides to enter will set its sample size equal to $N$. It is now easy to find the number of firms in a free-entry equilibrium. Since each firm's sample size is $N$, the net expected payoff to each firm is simply

$$E(\Pi_i) = \frac{gS}{n} - cN.$$

Entry will occur as long as $E(\Pi_i) > 0$, so that the equilibrium is given by

$$n_e = gS/cN.$$

In this scenario the optimal policy for a government-run research effort is clearly to have only one firm sampling the entire population, in which case the net social payoff is

$$W = S - cN.$$

Of course, if $S < cN$, it is best not to do any research; thus, it is assumed that $S/cN > 1$.

In principle, it is still possible for there to be underinvestment. In particular, if $g < cN/S < 1$, the competitive equilibrium will call for no research, while the social optimum calls for research spending of $cN$. If $g = cN/S$, the competitive equilibrium allocates just the right amount of resources to research, and it overinvests if $g > cN/S$.

## 4. The case of increasing costs

■ The model of Section 3 assumed constant average and marginal costs of doing research. There may be some question about the realism of this assumption. This section considers a version of the model where R&D is characterized by increasing marginal cost. This is consistent with the usual type of assumption that the size of cost reductions is an increasing, concave function of research spending.[13] It should be stressed here, however, that an argument may also be made for increasing returns to scale of R&D, since a firm may be better able to utilize information that it generates as the breadth or scope of its research activities is widened. The trouble with the latter type of situation is that it is difficult to identify a meaningful equilibrium.

It is now assumed that each firm chooses its sample size $y$, and that the R&D spending necessary is a function of $y$, $c(y)$ with

$$\lim_{y\to 0} c(y) > 0, \qquad c'(y) > 0, \qquad \text{and} \qquad c''(y) > 0.$$

The first assumption on the form of $c(y)$ is necessary to ensure an interior solution. With zero fixed costs of entry and $y$ truly variable, there would be an infinite number of entrants, each doing an infinitesimal amount of research. Of course, since $y$ is an integer, this is not feasible, and the possibility is avoided by the assumption here. It is assumed that all firms face the same cost function.

□ **The firm's problem.** The net expected payoff to a typical firm, say firm 1, is given by a slightly modified version of (10):

$$E(\Pi_1) = \frac{1}{N} y_1 g S V_1 - c(y_1), \tag{11}$$

where all notation is as before, but the cost term represents the new assumption on costs. The firm will choose $y_1$ to maximize $E(\Pi_1)$; therefore

$$\frac{\partial E(\Pi_1)}{\partial y_1} = \frac{1}{N} gSV_1 - c'(y_1) = 0. \tag{12}$$

Equation (12) yields a solution for $y_1$ as a function of the sample sizes of all other firms, which are implicit in $V_1$. There is a similar equation for each other firm that enters. Since firms are identical in all respects, I shall assume that their sample sizes are equal.[14] Further, under free entry, expected profits must be zero.

Now from (9) and the definition of $V_1$, we can write

$$V_1 = P(v = 0) + \tfrac{1}{2}P(v = 1) + \ldots + \frac{1}{n} P(v = n - 1).$$

Let $p = y/N$, where $y$ is the sample size of each firm, and let $q = 1 - p$. Then

$$V_1 = \sum_{i=0}^{n-1} \binom{n-1}{i} p^i q^{n-1-i} \left(\frac{1}{i+1}\right).$$

Multiplying and dividing the right-hand side by $np$ and simplifying, we may write

$$V_1 = \frac{N}{ny}(1 - q^n). \tag{13}$$

Using (13) in (11) and (12) and the fact that expected profits are zero, we find the equilibrium conditions for a symmetric free-entry equilibrium:

$$\frac{gS}{ny}(1 - q^n) = c'(y) \tag{14}$$

$$\frac{gS}{n}(1 - q^n) = c(y). \tag{15}$$

[13] This assumption has been made, for example, by Dasgupta and Stiglitz (1980b) and Nordhaus (1969), among others.

[14] Note that the symmetric equilibrium has been assumed, not derived from any primitive assumptions. For at least one demonstration that an R&D equilibrium may be asymmetric, see Flaherty (1980).

In principle, (14) and (15) together yield solutions for $y_e$ and $n_e$, the size of the typical firm and the number of firms in competitive equilibrium. Equation (14) is the first-order condition for firms' sample size choice and (15) is the zero-profit condition. Simplifying, we get

$$\frac{c(y)}{y} = c'(y). \tag{16}$$

Equation (16) simply states that the sample sizes of firms will be such that the average cost per research project is equal to the marginal cost. In other words, firms will produce research at minimum average cost. We shall return to discuss this result later. Note, however, that the sample size of each firm depends only on the R&D technology and not on the number of firms that have entered. Once $y_e$ is known, $n_e$ could be determined from (15).

□ **The social problem.** The social problem is to choose the number of firms and the sample size of each firm. For the sake of simplicity, we shall assume that the firms must have the same sample size. It is not clear that this must be so. In fact, an interesting theoretical problem would be to develop a model where the social optimum involves a diversity of firm R&D levels. Ignoring this for the moment, it is easy to show that the net expected social payoff is

$$E(W) = S(1 - q^n) - nc(y),$$

where all notation is as before. Note that this is the same as the expression derived for the basic model, equation (5), except that in (5) $c(y)$ was constant since $y$ was set equal to unity.

The necessary conditions for a maximum are

$$\frac{S}{N} q^{n-1} = c'(y) \tag{17}$$

$$-Sq^n \ln q = c(y). \tag{18}$$

Equations (17) and (18) together yield the solution values $y^*$ and $n^*$ for the optimal sample size and optimal number of firms.

□ **Comparison of the free-entry equilibrium and the socially optimal solutions.** Consider first the relative sample size of firms, i.e., compare $y_e$ and $y^*$.

*Proposition 2.* $y_e < y^*$. The sample size under free-entry equilibrium will always be too small as compared with the social optimum.

*Proof.* It has already been seen that firms in competition will perform R&D up to the point of minimum average cost, i.e., where average and marginal costs are equal. Consider now the relationship between average and marginal costs under the social optimum. At the optimum,

$$MC - AC = \frac{S}{N} q^{n-1} + \frac{Sq^n \ln q}{y}$$
$$= \frac{Sq^{n-1}}{y}(1 - q + q \ln q).$$

The term outside the parentheses is clearly positive, and, since $0 < q < 1$, it can be shown[15] that the expression in parentheses is also positive. Thus, at the social optimum,

[15] Let $X(q) = 1 - q + q \ln q$. Then $X(1) = 0$ and $X'(q) = \ln q < 0$. Thus $X(q)$ is monotonically decreasing for $0 < q < 1$ and falls to zero at $q = 1$. Therefore, $X(q) > 0$ for $0 < q < 1$.

marginal cost is greater than average cost. Since costs are convex, this implies

$$y^* > y_e.$$

The curious thing about this result is that normally we might expect the social optimum to involve producing at minimum average cost. Here, however, the competitive solution satisfies this requirement, and $y^*$ is unambiguously larger. The reason is that, at minimum average cost, social benefit is still increasing more rapidly than cost and *the same benefit cannot be obtained by replicating the existing firms.* By increasing the sample size of existing firms, society gains increased sampling *without replacement.* If it simply replicated existing firms, it would be increasing sampling with replacement. This makes it advantageous for society to increase the sample size beyond the minimum average cost point. The key element here is that two firms, each pursuing three projects, are not equivalent to one firm with six projects, since the two firms may simply be replicating each other's work. A firm that has already completed a particular research line is not going to spend any more resources on that line, but it is quite possible that another firm will. Of course, the advantage from sampling without replacement must be weighed against the increasing average cost of each research project.

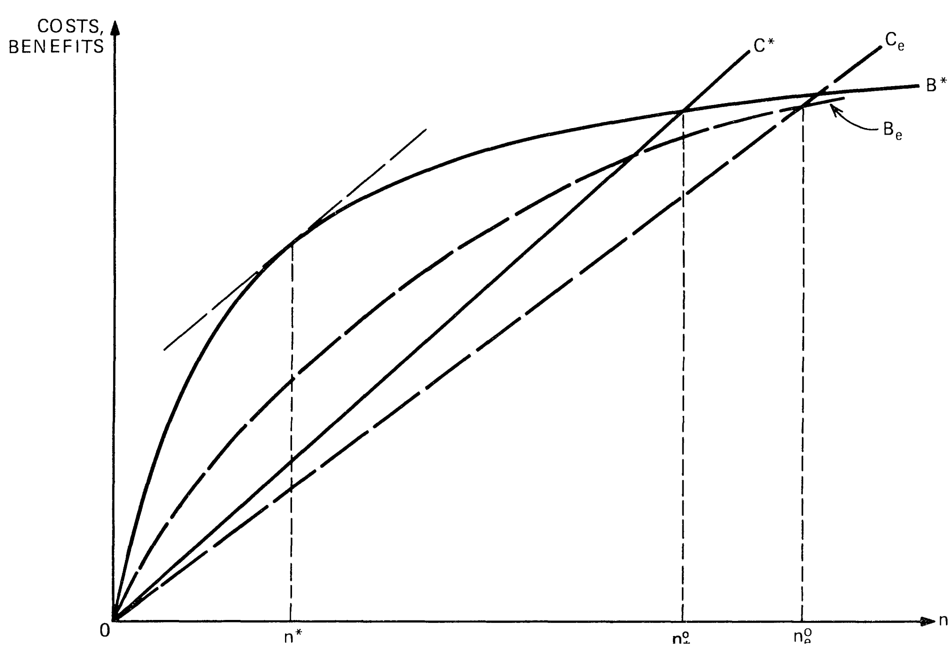
The next step is to compare the number of firms in the competitive and social solutions. Once again, it can be shown that there may be excessive or insufficient entry, depending on the degree of appropriability, $g$. One interesting question is whether the increasing costs strengthen or weaken the condition for overinvestment. Since the sample sizes of firms are too small compared with the social optimum, it might be argued that excessive entry would not be sustainable. The opposite turns out to be true, and the result is partially captured in the following proposition.

*Proposition 3.* Under full appropriability and increasing costs, the number of firms in competitive equilibrium will exceed the socially optimal number.

*Proof.* We know that $y^* > y_e$. Now consider Figure 2. The curves $B^*$ and $C^*$ represent social benefits and costs with optimal sample size. Thus,

FIGURE 2

SOCIAL AND PRIVATE SOLUTIONS WITH INCREASING COSTS



$$B^* = S(1 - q^n)$$

$$C^* = nc(y^*).$$

Both $B^*$ and $C^*$ are functions of $y$. (Recall that $q$ is a function of $y$.) Further, it is easy to show that

$$\frac{\partial B^*}{\partial y} > 0 \quad \text{and} \quad \frac{\partial C^*}{\partial y} > 0.$$

Let $n_*^o$ represent the value of $n$ at which $B^* = C^*$. Clearly $n_*^o > n^*$.

The curves $B_e$ and $C_e$ represent total benefits and costs when each firm's sample size is $y_e$. Under full appropriability, $n_e^o$ represents the number of firms in competitive equilibrium. It can be shown that $n_e^o > n_*^o$ and hence $n_e^o > n^*$.

Let $n_i^o$ represent the value of $n$ that equates social benefits with social costs for sample size $y_i$. Suppressing subscripts for simplicity, we have

$$S(1 - q^{n^o}) - n^o c(y) = 0.$$

The question is, how does $n^o$ change as $y$ falls from $y^*$ to $y_e$? By the implicit function theorem,

$$\frac{dn^o}{dy} = \frac{-n^o\left\{\frac{S}{N} q^{n^o - 1} - c'(y)\right\}}{-Sq^{n^o} \ln q - c(y)}.$$

The denominator is the social marginal benefit of $n$ minus the social marginal cost of $n$ which is always negative at $n^o$. The numerator is the negative of the marginal net social benefit of $y$. It is easy to show that social benefit is concave with respect to $y$ and costs are assumed convex in $y$, so that the marginal net social benefit of $y$ is a decreasing function of $y$. Further, at $y = 0$ both social benefit and social cost are zero. Thus, where benefit equals cost, marginal social benefit must be smaller than marginal cost. Therefore, the numerator is positive and $dn^o/dy < 0$. Thus, $n_e^o > n_*^o$ and hence $n_e^o > n^*$.

*Under full appropriability, the free-entry equilibrium is characterized by an excessive number of firms, each of which spends an insufficient amount on R&D.* This is similar in spirit to several results in the literature on monopolistic competition (Salop, 1979; Wolf, 1980), and sounds similar to the finding of Dasgupta and Stiglitz (1980b). With partial appropriability (i.e., $g < 1$), it is obvious that the result could go either way. With $g$ sufficiently small,[16] $n_e < n^*$. Thus, whether there is excessive entry depends crucially on the degree of appropriability.

What is the implication for total R&D spending and the amount of knowledge produced? The answer is ambiguous. There are several points to be noted here. First, it is not enough simply to compare levels of total spending if we are interested in the amount of knowledge production. It is now possible for there to be excessive spending and yet insufficient knowledge production, since firm sizes now differ between the free-entry and socially optimal solutions. Second, it is now possible for there to be an excessive number of firms and yet insufficient aggregate R&D. Third, there may still be excessive R&D spending and excessive knowledge production. This depends not only on the value of $g$, but also on the parameters of the cost function $c(y)$. Without further assumptions on functional forms, unambiguous results do not emerge. It is clear, however, that if $y_e$ is sufficiently "close" to $y^*$ and if $g$ is "large," the competitive equilibrium will continue to be characterized by excessive R&D spending and excessive knowledge production.

[16] We know this is true since, trivially, for $g = 0$, $n_e = 0$ and $n^*$ is positive.

## 5. Concluding remarks

This article has presented a simple framework for the analysis of research spending under conditions of free entry. Using a probability model, it was shown that free entry to the R&D game could lead to an excessive allocation of resources to research. One interesting feature of the results is that overinvestment of resources not only may reflect excessive duplication, but also may indicate overproduction of knowledge. In other words, the size of cost reductions under competition may be excessive. This could, under certain conditions, be interpreted as an argument in favor of increased concentration of research, which is consistent with a Schumpeterian view of technological change. But the reasoning here is actually the opposite of the traditional view. Specifically, the usual argument is that competition is preferable from the point of view of static efficiency, but that competitive firms will tend to produce insufficient cost reductions and hence that increased concentration is desirable for reasons of dynamic efficiency. Here I show that from the dynamic point of view, the competitive solution *may* produce cost reductions that are excessive, even though no one firm ever has the incentive to overproduce knowledge. Restricting the number of firms that do research might then be desirable, since the size of cost reductions would be reduced. Thus, our usual view of the "Schumpeterian tradeoff" may need some reassessment.[17] I should remind the reader, though, that I have not been very precise here about the nature of "competition." The question has been discussed, for example, by von Weizsäcker (1980).

What I have examined here are free-entry equilibria. Other types of equilibria could be examined: for example, cases with blocked entry, cartels, and so on. In some cases, the possibility of overinvestment disappears. In addition, another possible reason for excessive allocations has been ignored here—the possibility that firms may use R&D (and patents) as a barrier to entry.[18] Perhaps the most interesting line of enquiry that needs to be examined is the possibility of imitative research and its effect on incentives to spend on R&D. In particular, the public goods aspect of R&D needs to be more carefully modelled.

## References

ARDITTI, F. D. AND LEVY, H. "A Model of the Parallel Team Strategy in Product Development." *American Economic Review*, Vol. 70, No. 5 (December 1980), pp. 1089–1097.

ARROW, K. J. "Economic Welfare and the Allocation of Resources for Invention" in R. R. Nelson, ed., *The Rate and Direction of Inventive Activity*, Princeton: Princeton University Press, 1962.

BARZEL, Y. "Optimal Timing of Innovations." *Review of Economics and Statistics*, Vol. 50, No. 3 (August 1968), pp. 348–355.

BUCHANAN, J., TOLLISON, R., AND TULLOCK, G., Eds. *Towards a Theory of the Rent-Seeking Society*. College Station: Texas A & M University Press, 1980.

DASGUPTA, P. AND STIGLITZ, J. "Uncertainty, Industrial Structure, and the Speed of R&D." *Bell Journal of Economics*, Vol. 11, No. 1 (Spring 1980a), pp. 1–28.

——— AND ———. "Industrial Structure and the Nature of Innovative Activity." *Economic Journal*, Vol. 90, No. 358 (June 1980b), pp. 266–293.

FLAHERTY, M. T. "Industry Structure and Cost-Reducing Investment." *Econometrica*, Vol. 48, No. 5 (July 1980), pp. 1187–1209.

FUTIA, C. A. "Schumpeterian Competition." *Quarterly Journal of Economics*, Vol. 94, No. 4 (June 1980), pp. 675–695.

GALBRAITH, J. K. *American Capitalism*. Boston: Houghton Mifflin, 1956.

GORDON, H. S. "The Economic Theory of a Common Property Resource: The Fishery." *Journal of Political Economy*, Vol. 62, No. 2 (April 1954), pp. 124–142.

[17] See the recent paper by Nelson and Winter (1982). I have argued elsewhere (Tandon, 1982) that in fact on grounds of static efficiency a case can be made for restricting entry from the free-entry equilibrium. Thus, the Schumpeterian tradeoff might be completely reversed!

[18] For a discussion of this possibility, see von Weizsäcker (1980).

HIRSHLEIFER, J. "The Private and Social Value of Information and the Reward to Inventive Activity." *American Economic Review,* Vol. 61, No. 4 (September 1971), pp. 561–574.

KAMIEN, M. I. AND SCHWARTZ, N. L. "Timing of Innovations under Rivalry." *Econometrica,* Vol. 40, No. 1 (January 1972), pp. 43–60.

KITCH, E. W. "The Nature and Function of the Patent System." *Journal of Law and Economics,* Vol. 20, No. 2 (October 1977), pp. 265–290.

LEE, T. AND WILDE, L. L. "Market Structure and Innovation: A Reformulation." *Quarterly Journal of Economics,* Vol. 94, No. 2 (March 1980), pp. 429–436.

LEVIN, R. "Toward an Empirical Model of Schumpeterian Competition." National Bureau of Economic Research, Summer Institute Paper No. 80-11, December 1980.

LOURY, G. C. "Market Structure and Innovation." *Quarterly Journal of Economics,* Vol. 93, No. 3 (August 1979), pp. 395–410.

MARSCHAK, T., GLENNAN, T. K., AND SUMMERS, R. *Strategy for R&D: Studies in the Microeconomics of Development.* New York: Springer-Verlag, 1967.

NELSON, R. R. "Uncertainty, Learning, and the Economics of Parallel Research and Development Efforts." *Review of Economics and Statistics,* Vol. 43, No. 4 (November 1961), pp. 351–364.

———. "The Resource Allocation Problem When Innovation Is Possible." Yale University, Institution for Social and Policy Studies, Working Paper No. 820, May 1979.

——— AND WINTER, S. G. "Dynamic Competition and Technical Progress" in B. Balassa and R. R. Nelson, eds., *Economic Progress, Private Values, and Public Policy: Essays in Honor of William Fellner,* Amsterdam: North-Holland, 1977.

——— AND ———. "The Schumpeterian Tradeoff Revisited." *American Economic Review,* Vol. 72, No. 1 (March 1982), pp. 114–132.

NORDHAUS, W. D. *Invention, Growth and Welfare.* Cambridge: M.I.T. Press, 1969.

PETERSON, F. M. "Two Externalities in Petroleum Exploration" in G. M. Brannon, ed., *Studies in Energy Tax Policy,* Cambridge: Ballinger, 1975.

REINGANUM, J. F. "Dynamic Games of Innovation." *Journal of Economic Theory,* Vol. 25, No. 1 (August 1981), pp. 21–41.

ROBERTS, K. AND WEITZMAN, M. L. "Funding Criteria for Research, Development, and Exploration Projects." *Econometrica,* Vol. 49, No. 5 (September 1981), pp. 1261–1288.

ROSENBERG, N. "Science, Invention, and Economic Growth." *Economic Journal,* Vol. 84, No. 333 (March 1974), pp. 90–108.

SALOP, S. C. "Monopolistic Competition with Outside Goods." *Bell Journal of Economics,* Vol. 10, No. 1 (Spring 1979), pp. 141–156.

SCHMOOKLER, J. *Invention and Economic Growth.* Cambridge: Harvard University Press, 1966.

SCHUMPETER, J. *Capitalism, Socialism, and Democracy,* 2nd ed. London: Allen & Unwin, 1947.

SMITH, V. L. "Economics of Production from Natural Resources." *American Economic Review,* Vol. 58, No. 3 (June 1968), pp. 409–431.

———. "On Models of Commercial Fishing." *Journal of Political Economy,* Vol. 77, No. 2 (March/April 1969), pp. 181–198.

SPULBER, D. F. "Research and Development of a Backstop Energy Technology in a Growing Economy." *Energy Economics* (October 1980), pp. 199–207.

STIGLITZ, J. E. "The Efficiency of Market Prices in Long-Run Allocations in the Oil Industry" in G. M. Brannon, ed., *Studies in Energy Tax Policy,* Cambridge: Ballinger, 1975.

TANDON, P. "Aspects of Optimal R&D Policy." Unpublished Ph.D. dissertation, Harvard University, 1978.

———. "Innovation, Market Structure, and Welfare." Boston University Department of Economics Discussion Paper No. 83, June 1982.

WEITZMAN, M. L. "Free Access vs. Private Ownership as Alternative Systems for Managing Common Property." *Journal of Economic Theory,* Vol. 8, No. 2 (June 1974), pp. 225–234.

———. "Optimal Search for the Best Alternative." *Econometrica,* Vol. 47, No. 3 (May 1979), pp. 641–654.

VON WEIZSÄCKER, C. C. *Barriers to Entry.* Berlin: Springer-Verlag, 1980.

WOLF, R. G. "Regional Development, Monopsonistic Competition, and Public Enterprise." Boston University Department of Economics Discussion Paper No. 63, September 1980.